BINAR STAR SYSTEMS

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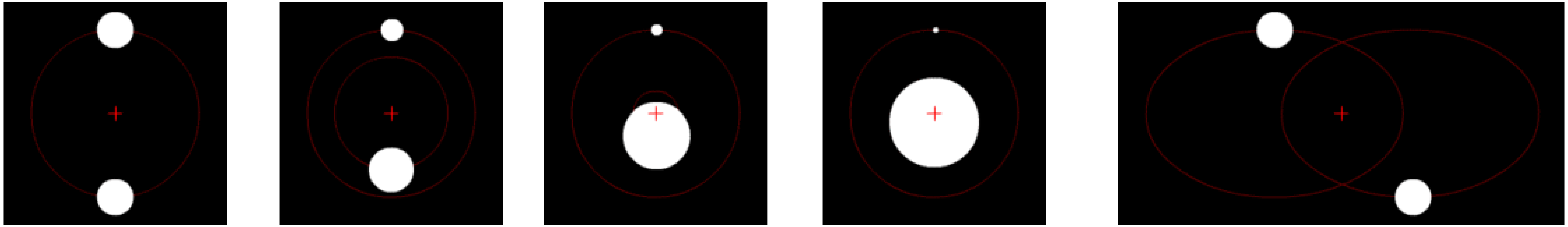
What is a binary star system?

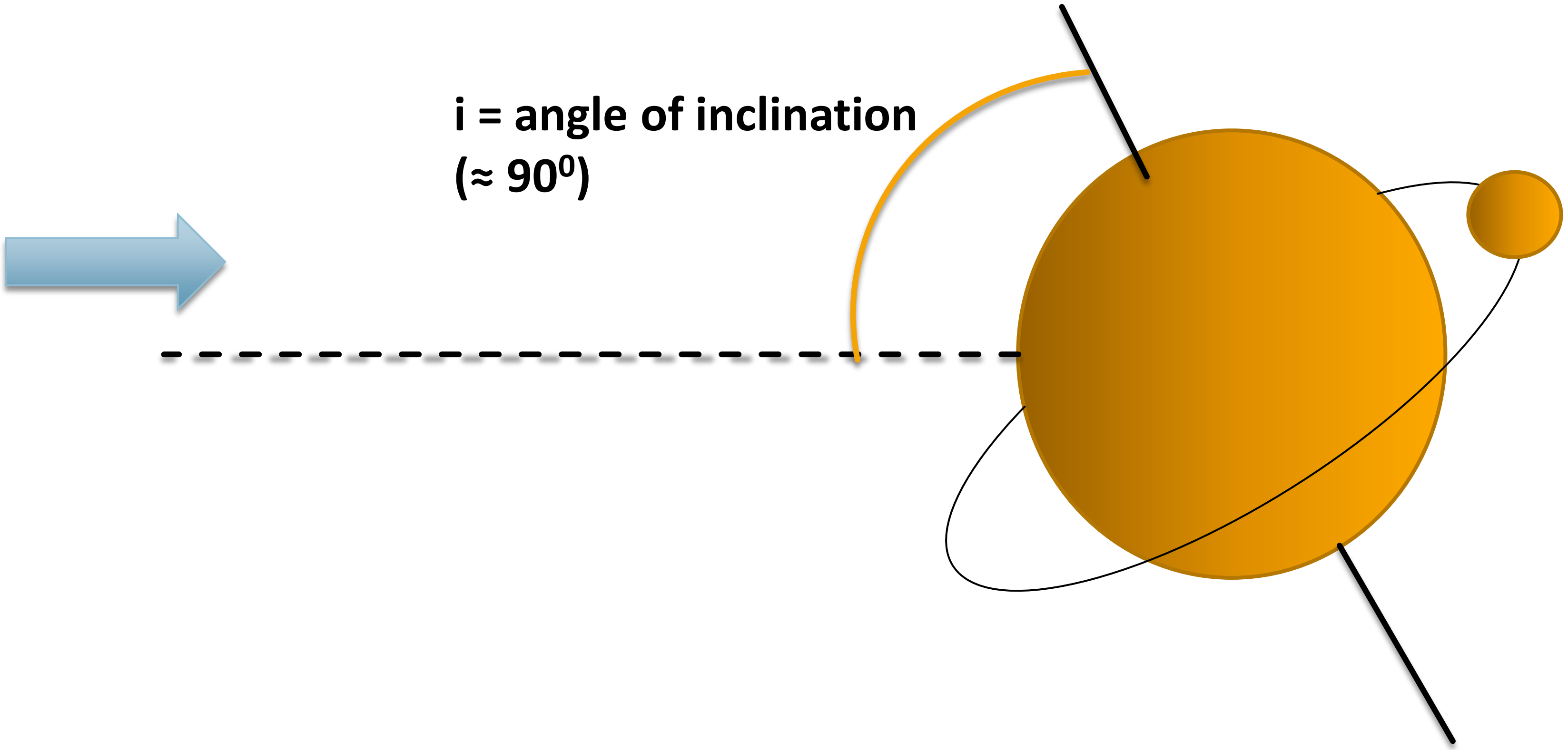
- A combination of two stars interacting with each other mainly through gravity



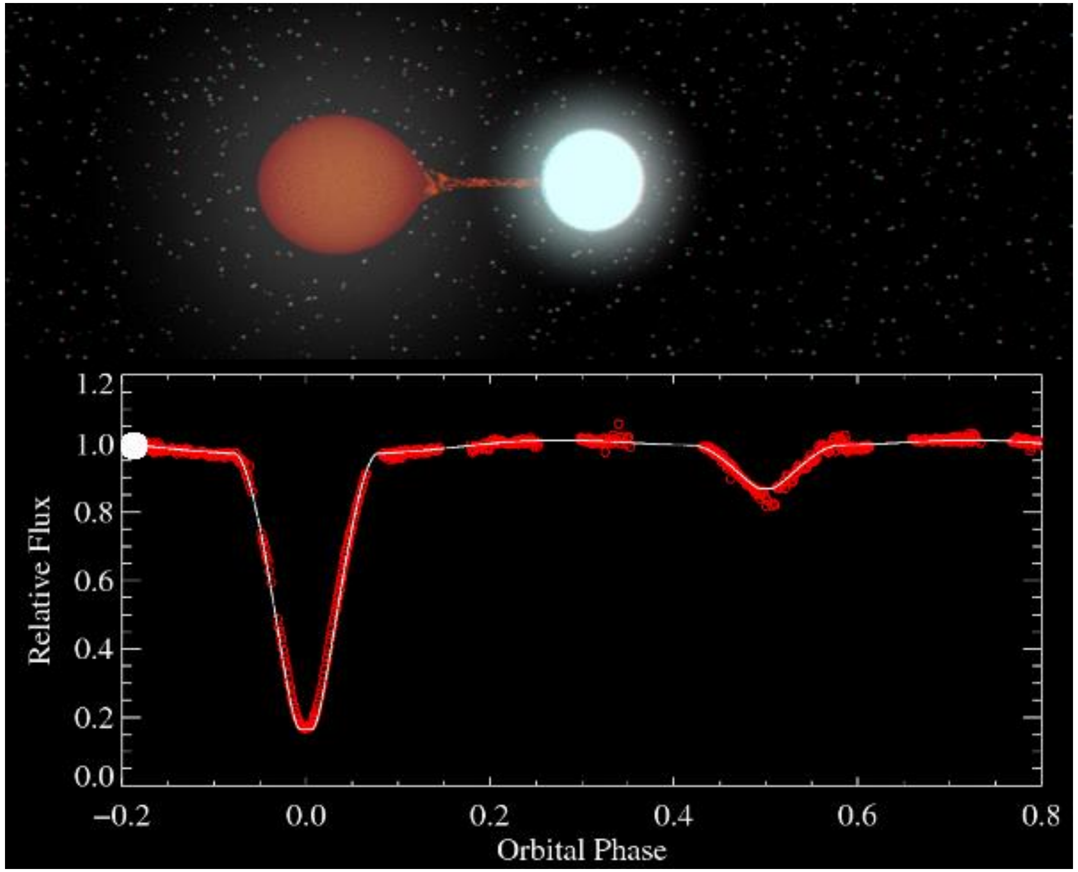
Types of Binary Star Systems

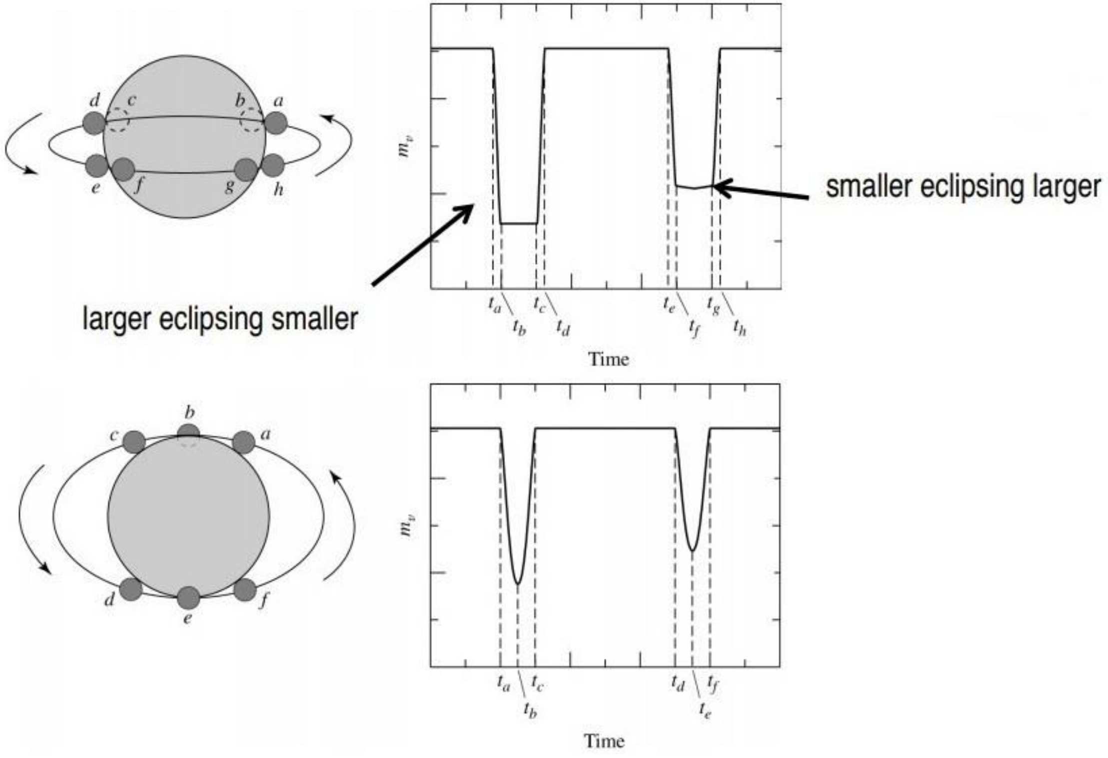
- Optical binary
- Visual binary
- **Eclipsing binary**
- Astrometric binary
- Spectroscopic binary
- Spectrum Binary
- Contact binary

Eclipsing Binary Star



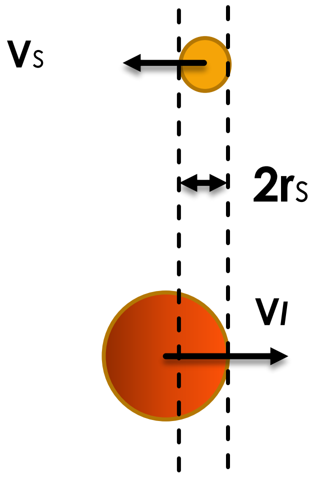
What we can see

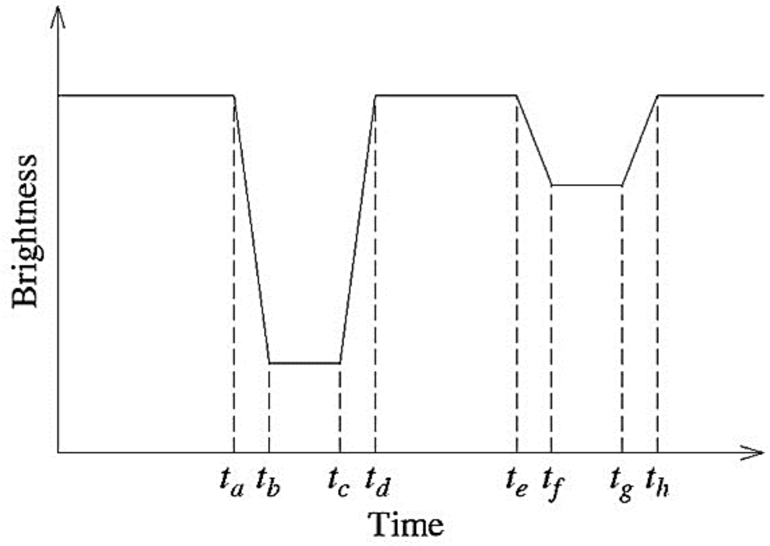




Once we know the **radius** of the orbit and the **period** of stars

Tangential velocities can be calculated





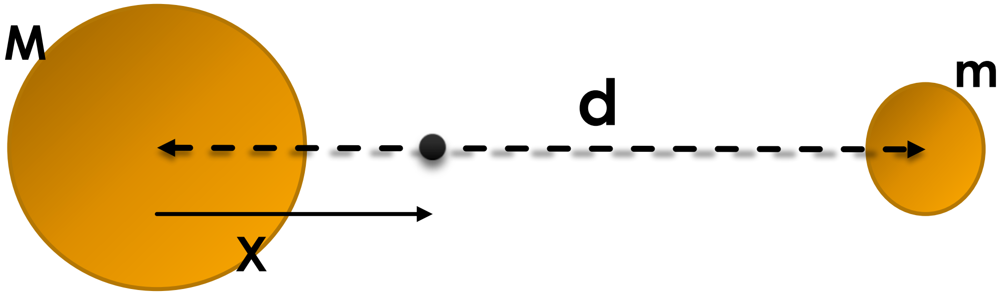
$V = V_l + V_s$

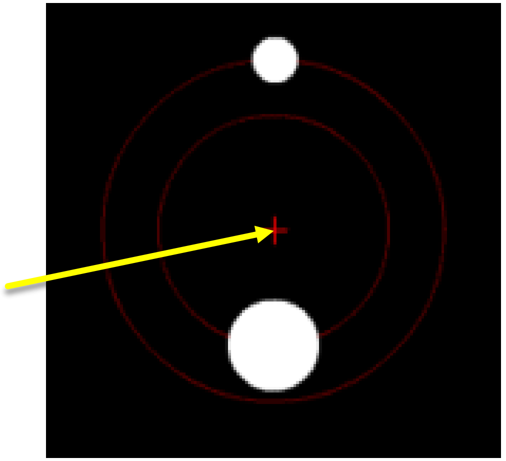
- $V = \frac{2r_s}{t_b - t_a}$
- $V = \frac{2r_l}{t_c - t_b}$

❖ r_s and r_l can be calculated

Barycenter

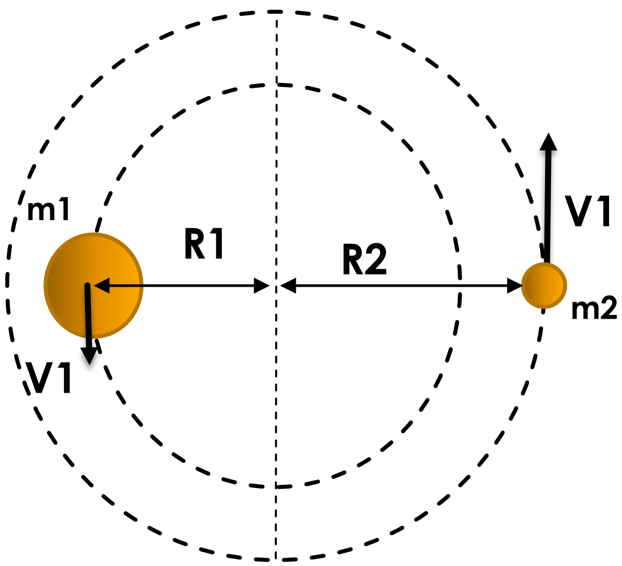
- The center of mass of a pair of astronomical objects. In other words both objects orbit around the barycenter





$$X = \frac{\sum md}{\sum m} = \frac{M(0) + md}{M + m} = \frac{md}{M + m}$$

In General



Let $R = R_1 + R_2$ and $m_1 = 3m_2$

$$X = R_1 = \frac{\sum md}{\sum m} = \frac{\boldsymbol{m1(0) + m2(R1 + R2)}}{\boldsymbol{m1 + m2}}$$

$$R1(m1 + m2) = m2(R1 + R2)$$

$$\cancel{R1m1 +} R1m2 = \cancel{R1m2 +} R2m2$$

$$\frac{\boldsymbol{m1}}{\boldsymbol{m2}} = \frac{\boldsymbol{R2}}{\boldsymbol{R1}} = \frac{\mathbf{3}}{\mathbf{1}}$$

Also by newton's second and third law,

$$F = ma = \frac{m_1 v_1^2}{R_1} = \frac{m_2 v_2^2}{R_2}$$

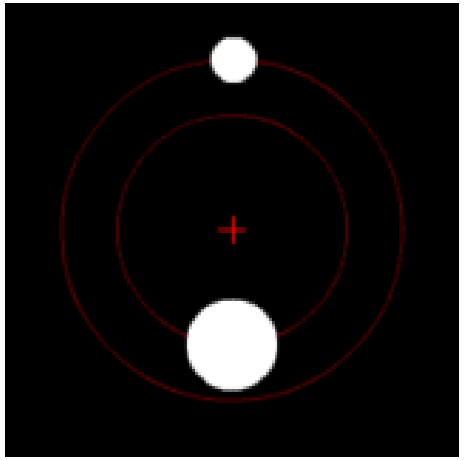
$$\frac{v_1^2}{v_2^2} = \frac{m_2}{m_1} \times \frac{R_1}{R_2}$$

$$\frac{v_1^2}{v_2^2} = \frac{m_2}{m_1} \times \frac{R_1}{R_2}$$

$$\frac{v_1^2}{v_2^2} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

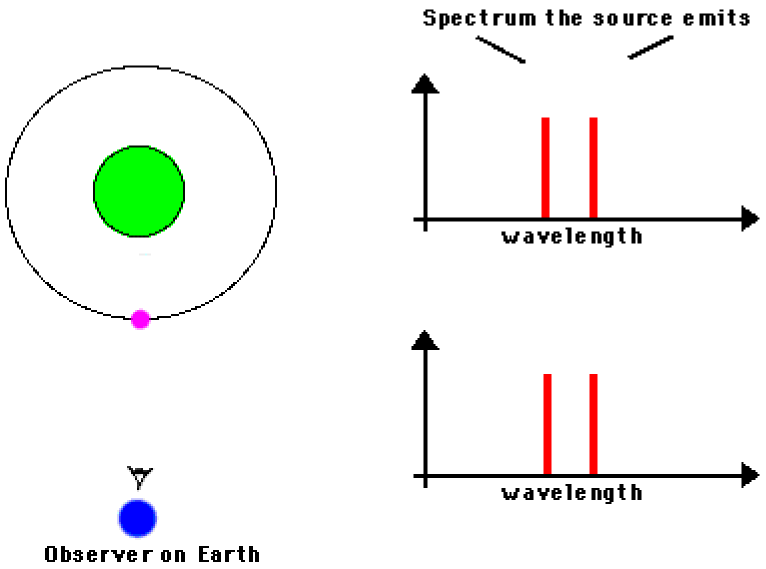
$$\boldsymbol{\frac{V_1}{V_2} = \frac{1}{3}}$$

$$\boldsymbol{\frac{m_1}{m_2} = \frac{R_2}{R_1} = \frac{v_2}{v_1} = 3}$$

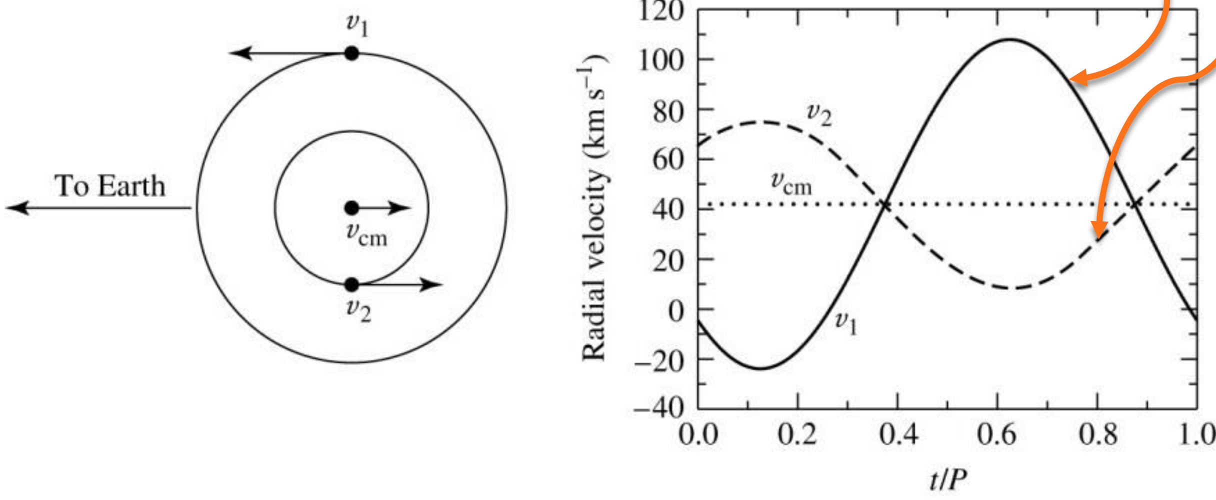


Radial Velocity Curves (Circular Orbits)

Using Doppler shift in the spectrums the radial velocities can be calculated

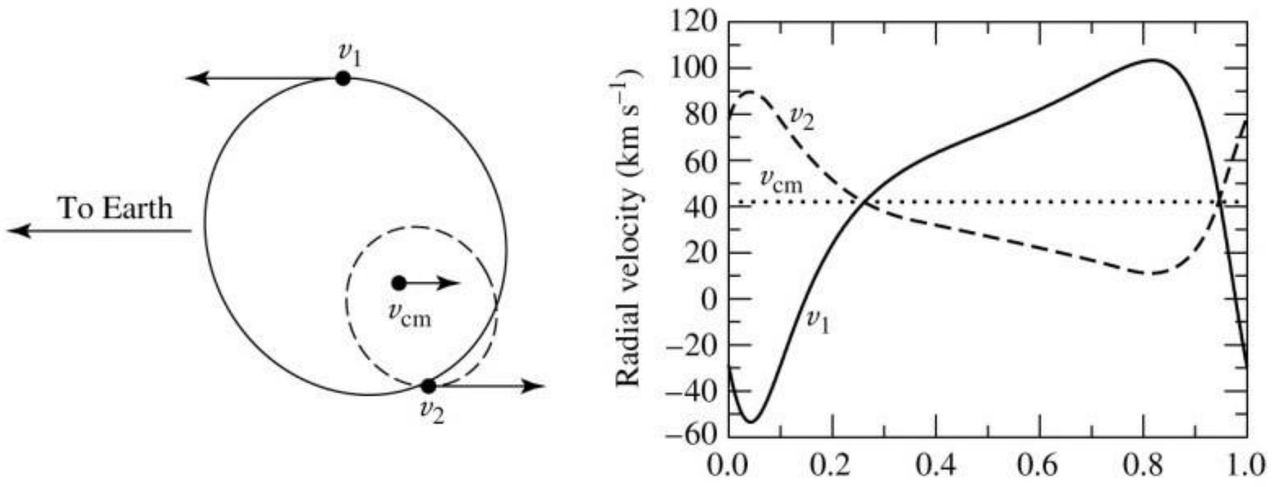


sinusoids



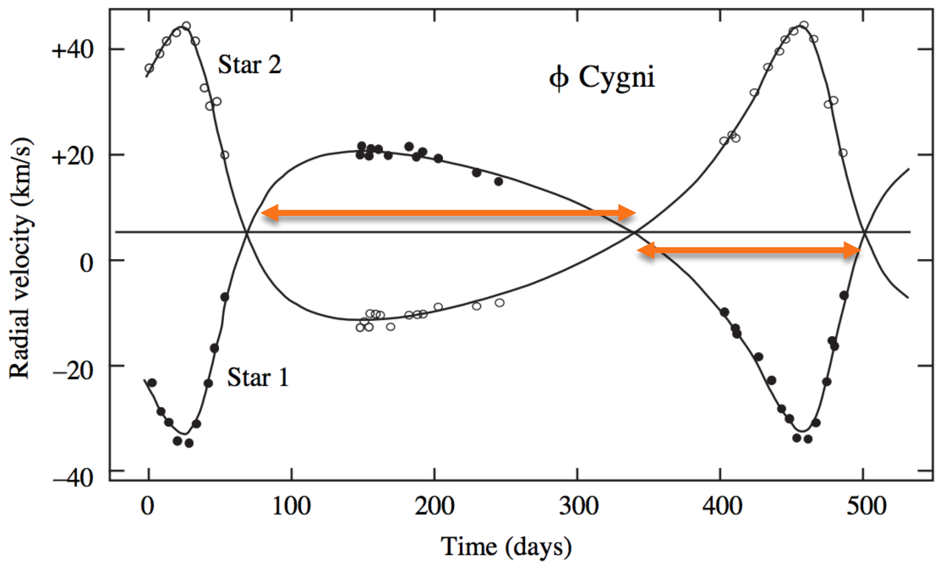
(a) (b)

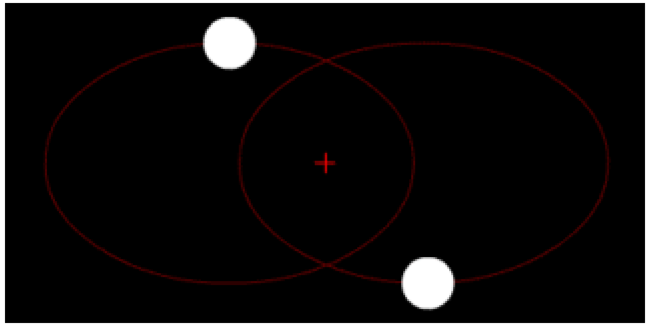
Non Circular Orbits



(a)

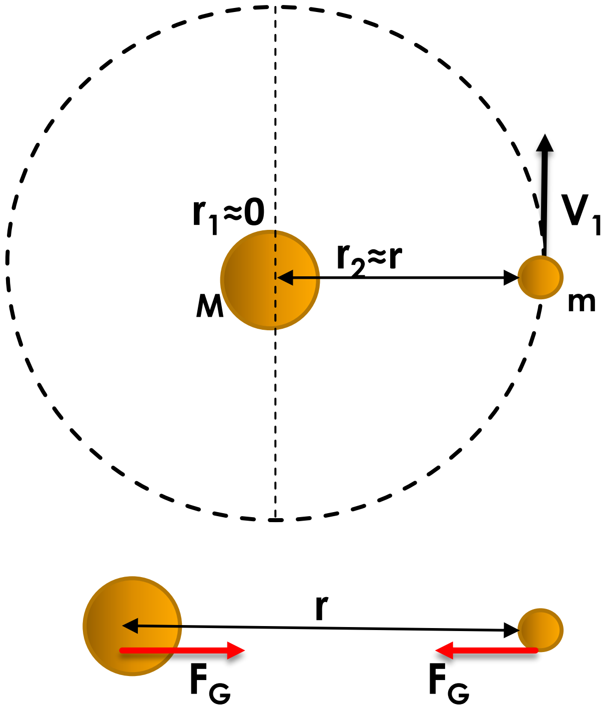
(b)





Determining the Masses

- **Example 1 : M >> m**



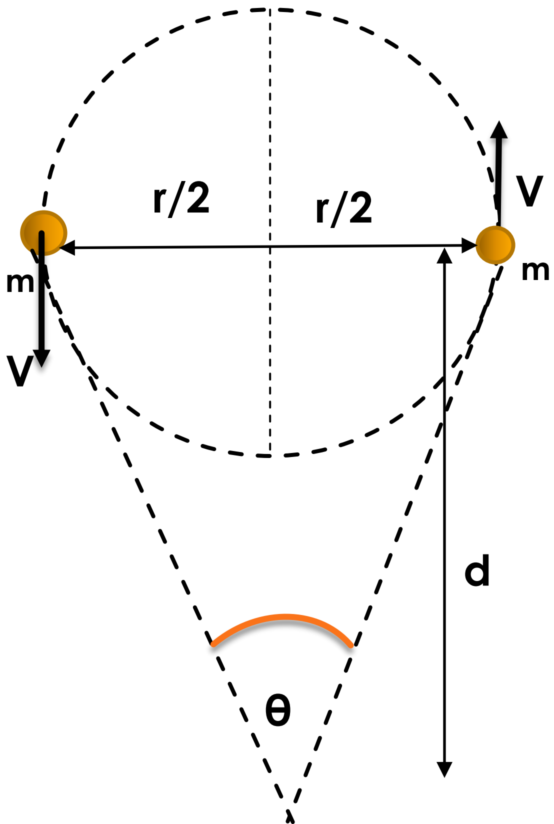
$\mathbf{r_1 \rightarrow 0}$ **and** $\mathbf{r_2 \approx r}$

Considering m

$$F_G = \frac{GMm}{r^2} = \frac{mv_1^2}{r}$$

- $\boldsymbol{M = \frac{v_1^2 r}{G}}$

- Example 2 : Two masses are equal $m_1 = m_2$ (d=10 ly, θ= 0.1arc seconds, $V_1=V_2=40kms^{-1}$)



If $\mathbf{m_1 = m_2}$ then $\mathbf{v_1 = v_2 = v}$ $\mathbf{r_1 = r_2 = r/2}$

$$F_G = \frac{Gmm}{r^2} = \frac{mv^2}{r/2}$$

$$\boldsymbol{m = \frac{2v^2 r}{G}}$$

But we don't know **r**

$$\tan\left(\frac{\theta}{2}\right) = \frac{r/2}{d}$$

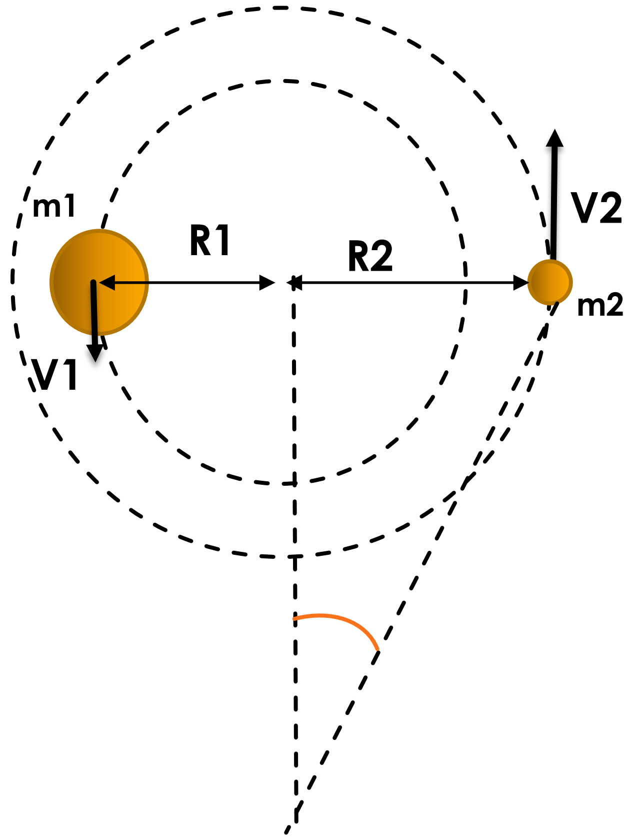
$$\tan\left(\frac{0.1}{3600}\right) = \frac{r/2}{10ly \times \frac{9.46 \times 10^{12} km}{ly}}$$

$$r = 62.6 \times 10^6 km$$

$$\boldsymbol{m = \frac{2 \times 40000^2 \times 62.6 \times 10^9}{6.67 \times 10^{-11}}}\ \mathbf{kg}$$

$$\underline{\boldsymbol{m = 3 \times 10^{30} kg}}$$

- Example 3 : Unequal masses (R=61.2 x 10^6 km, v_1=10kms^{-1}, v_2=30kms^{-1})



$$\frac{R_2}{R_1} = \frac{v_2}{v_1} = \frac{30kms-1}{10kms-1} = 3$$

$$R_2 = \frac{3}{4}R$$

$$F_G = \frac{Gm_1m_2}{R^2} = \frac{m_2v_2^2}{R_2}$$

$$\frac{Gm_1m_2}{R^2} = \frac{m_2v_2^2}{\frac{3R}{4}}$$

$$m_1 = \frac{4RV_2^2}{3G}$$

$$m_1 = \frac{4 \times 61.2 \times 10^9 \times 30000^2}{3 \times 6.67 \times 10^{-11}}$$

$$m_1 = 1.1 \times 10^{30} kg$$

Relationship With the period (P)

$F = \frac{Gm_1m_2}{r^2} = \frac{m_1v_1^2}{r_1}$, $m_1a_1 = m_2a_2$

$\frac{Gm_2}{a^2} = \frac{v_1^2}{a_1}$ ------(**1**)

But $V_1 = \frac{2\pi a_1}{P}$ ------**(2)**

$a = a_1 + a_2$

$a = a_1 + \frac{m_1a_1}{m_2} = \frac{a_1[m_1+m_2]}{m_2}$-------**(3)**

$$\frac{Gm_2}{a^2} = \frac{1}{a_1} \times \left(\frac{2\pi a_1}{P}\right)^2 = \frac{4\pi^2}{p^2} \times \frac{am_2}{(m_1+m_2)}$$

$$\boldsymbol{P^2 = \frac{4\pi^2 \times a^3}{G(m_1+m_2)}}$$